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Selective Measures under Constant and Variable Returns-To-Scale Assumptions

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Abstract**:** *Performance assessment of firms or companies with similar production process is an important issue in management and economics. Data envelopment analysis (DEA) is a data-based methodology that evaluates firms directly with the observed performance measures. In the DEA approach, the relative efficiency of each firm is measured by solving a mathematical programming model and this model discriminates firms to efficient and inefficient units. One of the critical issues in this process accurses when the number of firms is less than the number of performance measures. In this situation, a large number of firms are evaluated as efficient and hence the obtained results are questionable. One method to deal with this issue is selecting some performance measures which are named selective measures. In this paper, we compare two individual and aggregate selecting approaches under constant and variable returns-to-scale assumptions. A real dataset involving 14 active banks in the Czech Republic is utilized to illustrate how these approaches opt selective measures.*

Keywords: *Performance evaluation, data envelopment analysis; selective measures; multiplier form selecting models; baking industry.*

1. Introduction

Data envelopment analysis (DEA) is a well-organized optimization-based methodology to resolve and analyze the problem of performance assessment of n homogeneous decision making units (DMUs), which consume m inputs to produce s outputs. In the basic DEA models, performance measures (inputs and outputs) are summarized in a number named relative efficiency. The relative efficiency of a DMU is computed by maximizing the ratio of weighted sum of its outputs to weighted sum of its inputs, based on the condition that this ratio is less than or equal to one for all DMUs. To compute the relative efficiency of each DMU Charnes, et al. [1] in their pioneering work formulated a fractional programming and then reformulated and solved this fractional programming as a linear programming problem. Along with the speedy advances in linear programming and operations research, DEA models have been also rapidly developed. In the course of this development, some critical challenges have occurred [2]. One of these challenges occurs when the number of performance measures is high in comparison with the number of DMUs; in this situation, most of the DMUs are evaluated efficient and hence the obtained results are not reliable [3]. On the other hand, awkward removing of some measures from considerations can extremely affect the efficiency status of DMUs.

Empirically, there is a rough rule of thumb [4], which expresses the relation between the number of DMUs and the number of performance measures:

$$n \geq \max\{3(m+s), m \times s\} \tag{1}$$

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In some real-world problems, the number of performance measures and the number of DMUs do not satisfy the rule of thumb. In such situations, selecting a number of appropriate measures, in a manner which fulfills (1) is an important issue. A variety of researchers attempted to tackle this issue: Toloo et al. [3] proposed two individual DMU and aggregate models to develop the idea of selective measures, under the constant returns-to-scale (CRS) assumption. The authors modified the multiplier and envelopment forms of CRS model to obtain an approach that selects a set of performance measures so that the number of DMUs, inputs, and outputs meet the rule of thumb.Toloo and Tichý [5] improved Toloo et al.'s [3] models with the aim of developing a pair of alternative approaches for selecting performance measures under variable returns-to-scale (VRS) assumption. The authors proved that the multiplier approach leads to the maximum efficiency scores whiles the maximum discrimination between efficient units is achieved by applying the envelopment approach. Keshavarz and Toloo [6] proposed a single stage approach for selecting inputs/outputs in DEA, based on the VRS assumption and the common set of weights methodology. Toloo and Allahyar [7] extended an envelopment form of selecting the model of Toloo et al. [3].

The current study makes a comparison between two individual and aggregate multiplier forms of selecting models under CRS and VRS assumption. The rest of the paper is organized as follows: Section 2 provides a review of the standard DEA models including the multiplier forms of the CCR and BCC models. In Sections 3, we review two individual-based selecting models under CRS and VRS assumptions. A pair of aggregate models is formulated in Section 4 to select the adequate performance measures for CRS and VRS technologies. A real data set of bank industry is used to illustrate the applicability and comparison of proposed models, in Section 5. Conclusions and future researchers are provided in the last section.

2. Standard DEA models

Charnes et al. [1] proposed an outstanding mathematical programming model for evaluating the relative efficiencies among DMUs with multi-input and multi-output. This model, which is referred to as CCR (Charnes, Cooper, and Rhodes), assumed CRS technology. Banker et al. [8] suggested a new model, which is known as BCC (Banker, Charnes, and Cooper), to deal with VRS situation.

Consider the problem of evaluating a set of n DMUs, each consuming various amounts of m inputs to produce s outputs. Let $\boldsymbol{x}_j = (x_{1j},\ldots,x_{mj})$ and $\boldsymbol{y}_j = (y_{1j},\ldots,y_{sj})$ represent the input and output vectors for DMU_j $(j = 1,\ldots,n)$, respectively. Models (2) and (3) describe the CCR and BCC models, respectively.

$$
\begin{aligned}
& \max \theta_o = \textstyle\sum_{r=1}^{s} u_r y_{ro} \\
& \text{s.t.} \\
& \textstyle\sum_{i=1}^{m} v_i x_{io} = 1 \\
& \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad \forall j \\
& v_i, u_r \geq \varepsilon \qquad \forall i, \forall r
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
& \max \theta_o = \textstyle\sum_{r=1}^{s} u_r y_{ro} + u_0 \\
& \text{s.t.} \\
& \textstyle\sum_{i=1}^{m} v_i x_{io} = 1 \\
& \textstyle\sum_{r=1}^{s} u_r y_{rj} + u_0 - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad \forall j \\
& v_i, u_r \geq \varepsilon \qquad \forall i, \forall r \\
& u_0 \text{ free is sign}
\end{aligned} \tag{3}
$$

where u_r and v_i are the set of output and input weights, respectively, u_o is a variable with free in sign; and $\varepsilon > 0$ is the non-Archimedean infinitesimal, which is employed to hinder the weights to become zero [9]. The optimal value of θ_o in models (2) and (3) is named CCR and BCC-efficiency score of DMU_o, under CRS and VRS assumptions, respectively. If $\theta_o^* = 1$, then DMU_o is efficient and otherwise is inefficient. It is easy to show that CCR-efficiency score of each DMU is less than or equal to its BCC-efficiency score [4].

Values of efficiency scores are appropriate criteria to discriminate and rank all DMUs for the managerial aims. Nevertheless, it is possible that there exist some cases which the number of performance measures and DMUs contravene the rule of thumb and discriminating power of models (2) and (3) reduced. In the next section, we present selecting DEA models which are proposed by Toloo et al. [3] and Toloo and Tichý [5] which can handle the situations.

3. Selective Measures under CRS and VRS assumptions

Consider n DMUs, with m inputs and s outputs, where the rule of thumb is violated i.e. $n < max\{3(m+s), m \times s\}$. In order to having a sharper discrimination among DMUs logical selecting of performance measures, such that the rule of thumb is met, is a problem that needs to be solved. Let s_1 and s_2 denote subsets of outputs corresponding to fixed-output and selective-output measures, respectively. Similarly, assume that m_1 and m_2 are the parallel subsets of inputs. Following Toloo et al. [3] and Toloo and Tichý [5] we propose the following mixed integer programming models to select the number of performance measures as individual CRS and VRS selecting models.

$$\begin{array}{ll}
\max \sum_{r=1}^{s} u_r y_{ro} & \\
\text{s.t.} & \\
\sum_{i=1}^{m} v_i x_{io} = 1 & \\
\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 & \forall j \\
\sum_{r \in s_2} b_r^y = p & \\
\sum_{i \in m_2} b_i^x = q & \\
\varepsilon b_i^x \leq v_i \leq M b_i^x & \forall i \in m_2 \\
\varepsilon b_r^y \leq u_r \leq M b_r^y & \forall r \in s_2 \\
b_i^x, b_r^y \in \{0,1\} & \forall i \in m_2, \forall r \in s_2 \\
v_i, u_r \geq \varepsilon & \forall i \in m_1, \forall r \in s_1
\end{array} \tag{4}$$

$$\begin{array}{ll}
\max \sum_{r=1}^{s} u_r y_{ro} + u_0 & \\
\text{s.t.} & \\
\sum_{i=1}^{m} v_i x_{io} = 1 & \\
\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \leq 0 & \forall j \\
\sum_{r \in s_2} b_r^y = p & \\
\sum_{i \in m_2} b_i^x = q & \\
\varepsilon b_i^x \leq v_i \leq M b_i^x & \forall i \in m_2 \\
\varepsilon b_r^y \leq u_r \leq M b_r^y & \forall r \in s_2 \\
b_i^x, b_r^y \in \{0,1\} & \forall i \in m_2, \forall r \in s_2 \\
v_i, u_r \geq \varepsilon & \forall i \in m_1, \forall r \in s_1
\end{array} \tag{5}$$

where $\varepsilon > 0$ is a non-Archimedean infinitesimal and M is a large positive number. Binary variables b_i^x and b_r^y are associated with selective input $i \in m_2$ and selective output $r \in s_2$, respectively. It should be noted that b_i^x is equal to 1 if its associated input is selected, and the same result can be achieved for b_r^y. Parameters p and q are natural numbers and represent the number of selected inputs and outputs, respectively. Following theorem shows the condition that models (4) and (5) comply the rule of thumb.

Theorem 1. The presented models (4) and (5) will meet the rule of thumb if $p + q \leq \min\left\{\left[\frac{n}{3}\right], 2\sqrt{n}\right\} - (|m_1| + |s_1|)$.

Proof. We prove the theorem for model (4), the correctness theorem for model (5) is proved in the same manner.

Let $(\boldsymbol{u}^*, \boldsymbol{v}^*, \mathbf{b}^{y*}, \mathbf{b}^{x*})$ be the optimal solution of selecting model (4). It is clear that if $b_i^{x*} = 1$, then the i^{th} selective input is selected. In a similar manner, the r^{th} selective output is selected when $b_r^{y*} = 1$. Hence, the total number of involved inputs and output, including fixed and selective measures is equal to $\left(|m_1| + \sum_{i\in m_2} b_i^{x^*}\right) + \left(|s_1| + \sum_{r\in s_2} b_r^{y^*}\right)$. Now, taking the constraint $\sum_{r\in s_2} b_r^y + \sum_{i\in m_2} b_i^x = p + q \le \min\{[n/3], 2\sqrt{n}\} - (|m_1| + |s_1|)$ into consideration, two cases may arise:

(i) $[n/3] = \min\{[n/3], 2\sqrt{n}\}$ which implies $n \ge 3\left(\left(|m_1| + \sum_{i\in m_2} b_i^{x^*}\right) + \left(|s_1| + \sum_{r\in s_2} b_r^{y^*}\right)\right)$

(ii) $2\sqrt{n} = min\{[n/3], 2\sqrt{n}\}$ which considering the constraint $\sum_{r\in s_2} b_r^y + \sum_{i\in m_2} b_i^x \le 2\sqrt{n} - (|m_1| + |s_1|)$ leads to $n \ge \left(|m_1| + \sum_{i\in m_2} b_i^{x^*}\right) \times \left(|s_1| + \sum_{r\in s_2} b_r^{y^*}\right)$.

As a result, from the optimal solution of the model (4) we obtain $n \ge \max\left\{3\left(\left(|m_1| + \sum_{i\in m_2} b_i^{x^*}\right) + \left(|s_1| + \sum_{r\in s_2} b_r^{y^*}\right)\right), \left(\left(|m_1| + \sum_{i\in m_2} b_i^{x^*}\right)\left(|s_1| + \sum_{r\in s_2} b_r^{y^*}\right)\right)\right\}$ which completes the proof. ■

Models (4) and (5) select performance measures and finds their weights that are DMU-specific, and it is acceptable for individual circumstances of operation of DMUs, in practice, it can fail to discriminate on the performance of DMUs. Indeed, efficiency of DMU_o under-evaluation is measured from an optimistic point of view and this DMU attains the maximum or near maximum score. On the other hand, there are some circumstances that different DMUs are controlled and evaluated under management of a top-manager and therefore, utilizing different performance measures with different weights for DMUs may not be acceptable. When that is the case, both the inputs and the outputs should be aggregated by using weights that are common to all the DMUs [10]. In the next section, two aggregate models will be introduced to select the performance measures and find common set of weights.

4. Aggregate Selecting Models

The overall performance of the collection of DMUs can be considered as an alternative approach for selecting performance measures. To monitor this aspect, we study the performance efficiency of the aggregate outputs to aggregate inputs with the outlook on selective measures. To this end, we formulate two following MIPs to obtain an aggregate efficiency, under CRS and VRS assumptions, respectively.

$$\begin{aligned}
&\max \textstyle\sum_{r=1}^{s} u_r\left(\sum_{j=1}^{n} y_{rj}\right) \\
&\text{s.t.} \\
&\textstyle\sum_{i=1}^{m} v_i \sum_{j=1}^{n} x_{ij} = 1 \\
&\textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 && \forall j \\
&\textstyle\sum_{r\in s_2} b_r^y = p \\
&\textstyle\sum_{i\in m_2} b_i^x = q \\
&\varepsilon b_i^x \le v_i \le M b_i^x && \forall i \in m_2 \\
&\varepsilon b_r^y \le u_r \le M b_r^y && \forall r \in s_2 \\
&b_i^x, b_r^y \in \{0,1\} && \forall i \in m_2, \forall r \in s_2 \\
&v_i, u_r \ge \varepsilon && \forall i \in m_1, \forall r \in s_1
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\max \textstyle\sum_{r=1}^{s} u_r\left(\sum_{j=1}^{n} y_{rj}\right) + n u_0 \\
&\text{s.t.} \\
&\textstyle\sum_{i=1}^{m} v_i \sum_{j=1}^{n} x_{ij} = 1 \\
&\textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0 && \forall j \\
&\textstyle\sum_{r\in s_2} b_r^y = p \\
&\textstyle\sum_{i\in m_2} b_i^x = q \\
&\varepsilon b_i^x \le v_i \le M b_i^x && \forall i \in m_2 \\
&\varepsilon b_r^y \le u_r \le M b_r^y && \forall r \in s_2 \\
&b_i^x, b_r^y \in \{0,1\} && \forall i \in m_2, \forall r \in s_2 \\
&v_i, u_r \ge \varepsilon && \forall i \in m_1, \forall r \in s_1
\end{aligned} \tag{7}$$

In the next section, a real dataset of banking industry in the Czech Republic is used to illustrate the applicability of the proposed models and compare their results.

5. Case Study

In this section, we use a real dataset obtained from the annual reports of 14 active banks in the Czech Republic in order to compare the selected performance measures with individual and aggregate approaches under CRS and VRS assumptions. Table 1 demonstrates the data set of 14 banks including five inputs, Employees (x_1), Number of branches (x_2), Assets (x_3), Equity (x_4), and Expenses (x_5), along with four outputs (Deposits (y_1), Loans (y_2), Non-interest income (y_3), Interest income (y_4). In this study, we assume that all inputs and outputs are selective measures and the models should select some measures among all performance measures.

Table I: Demonstrates the data set of 14 banks including five inputs and outputs

Banks	*Inputs*					*Outputs*				CCR-efficiency	BCC-efficiency
	(x_1)	(x_2)	(x_3)	(x_4)	(x_5)	(y_1)	(y_2)	(y_3)	(y_4)		
AIR	400	18	33600	2596	745	30696	11135	14	554	1	1
CMZRB	217	5	111706	4958	566	86967	16813	634	1700	1	1
CS	10760	658	920403	93190	18259	688624	489103	15412	37717	1	1
CSOB	7801	322	937174	73930	16087	629622	479516	8747	32697	1	1
EQB	296	13	8985	1296	601	7502	5611	19	215	1	1
ERB	72	1	33614	464	173	2940	1762	15	131	0.4737	1
FIO	59	36	18561	726	347	17174	6465	211	536	1	1
GEMB	3346	260	135474	34486	5276	97063	101898	3943	11026	1	1
ING	293	10	128425	913	1034	92579	19216	468	5139	1	1
JTB	407	3	85087	7233	1333	62085	39330	487	3686	1	1
KB	8758	399	786836	100577	13511	579067	451547	8834	35972	1	1
LBBW	365	18	31300	2774	1138	20274	2528	128	1046	0.8246	0.8625
RB	2927	125	197628	18151	57112	144143	150138	2829	8563	1	1
UCB	2004	98	318909	38937	13804	195120	192046	2740	8891	1	1

The last two columns in Table 1 shows that 86% of banks (i.e. 12 out of 14) are CCR-efficient and 93% of banks (i.e. 13 out of 14) are BCC-efficient. These results are questionable because the large number of performance measures exists in comparison with the number of DMUs ($14 = n < 27 = \max\{3(m+s), m \times s\}$. To obtain acceptable result, we have to select the number of performance measure such that the rule of thumb is satisfied, i.e. n excess $3(m + s)$. For this purpose, we first solve the individual-based selecting models (4) and (5), by assuming $p = 2$ and $q = 2$, as a managerial suggestion which satisfies the condition of Theorem 1.

Table II: Results of solving models (4) and (5) for the dataset

Banks	Model (4)									Model (5)								
	b_i^x					b_r^y				b_i^x					b_r^y			
	(x_1)	(x_2)	(x_3)	(x_4)	(x_5)	(y_1)	(y_2)	(y_3)	(y_4)	(x_1)	(x_2)	(x_3)	(x_4)	(x_5)	(y_1)	(y_2)	(y_3)	(y_4)
AIR	0	1	1	0	0	1	0	0	1	0	1	1	0	0	1	1	0	0
CMZRB	0	1	0	0	1	1	1	0	0	1	1	0	0	0	1	1	0	0
CS	0	0	1	0	1	1	0	1	0	1	1	0	0	0	1	0	0	1
CSOB	0	0	0	1	1	1	1	0	0	1	1	0	0	0	1	1	0	0
EQB	0	0	1	0	1	1	1	0	0	0	1	1	0	0	1	1	0	0
ERB	0	1	0	1	0	0	1	1	0	0	1	0	1	0	1	1	0	0
FIO	0	1	1	0	0	1	0	1	0	1	1	0	0	0	1	1	0	0
GEMB	0	1	1	0	0	1	0	1	0	0	1	1	0	0	1	0	1	0
ING	0	1	0	1	0	1	1	0	0	1	1	0	0	0	1	1	0	0
JTB	1	1	0	0	0	1	1	0	0	1	1	0	0	0	1	1	0	0
KB	0	1	0	0	1	0	1	1	0	0	1	0	0	1	1	1	0	0
LBBW	0	1	1	0	0	1	0	0	1	0	1	1	0	0	1	0	0	1
RB	0	1	1	0	0	0	1	1	0	0	1	0	1	0	1	1	0	0
UCB	1	0	1	0	0	1	1	0	0	1	1	0	0	0	0	1	1	0
Sum	**2**	**10**	**8**	**3**	**5**	**11**	**9**	**6**	**2**	**7**	**14**	**4**	**2**	**1**	**13**	**11**	**2**	**2**

Table II shows the optimal values of binary variables b_i^x $(i = 1, \dots, 5)$ and b_r^y $(r = 1, \dots, 4)$; these variables characterize selected measures. As can be seen, x_2, x_3, y_1, and y_2 which have maximum frequency are selected measures in CRS individual model, also x_1, x_2, y_1, and y_2 have the same position in VRS individual model. Both individual-based models select three common selective measures, i.e. x_2, y_1, and y_2. The first two columns of Table 3 show the efficiency scores of all banks in the presence of these selected measures. As it can be seen, the percentage of CCR- and BCC-efficient banks is reduced to 50% and 57%, respectively.

In order to do an adaptive comparison, we solve the aggregate selecting models (6) and (7). The results point out that model (6) selects x_1, x_4, y_2, and y_3 measures meanwhile model (7) identifies selects x_2, x_4, y_1, and y_3 measures. It should be noted that, both CRS and VRS aggregate approaches opt x_4 and y_3 measures. The last two columns of Table 3 exhibit the efficiency scores of all banks obtained under CRS and VRS technologies in the presence of the selected measures. As we expected, the percentage CCR- and BCC-efficient DMUs via the aggregated-based approaches is decreased to 14% and 43% which illustrate the discriminating power of aggregate-based approach over the individual-based approaches.

TABLE III: Efficiency Scores Obtained by Individual And Aggregate Approaches

Banks	Individual-based approach		Aggregate-based approach	
	CCR-efficiency	BCC-efficiency	CCR-efficiency	BCC-efficiency
AIR	1.0000	0.3761	0.3406	0.2323
CMZRB	1.0000	1.0000	0.8170	1.0000
CS	0.9188	1.0000	0.5131	1.0000
CSOB	0.8948	1.0000	0.6401	1.0000
EQB	1.0000	0.3419	0.2647	0.3757
ERB	0.1421	1.0000	0.3002	1.0000
FIO	1.0000	1.0000	1.0000	0.9067
GEMB	0.9901	0.3166	0.3746	0.6487
ING	0.9154	1.0000	1.0000	1.0000
JTB	1.0000	1.0000	0.8819	1.0000
KB	0.9501	0.8263	0.4880	0.8180
LBBW	0.7088	0.2701	0.1368	0.2076
RB	1.0000	0.5754	0.6384	0.8822
UCB	1.0000	1.0000	0.8746	0.8907

6. Conclusion and Future Researchers

This paper deals with selective measures in DEA with the aim of drawing a comparison between both individual- and aggregate-based approaches under CRS and VRS assumptions. Our results point out that the number of efficient DMUs is significantly decreased via both individual- and aggregate-based approaches under CRS and VRS technologies. It is also shown that different selective measures might be selected by considering different types of returns-to-scale. A case study is utilized to illustrate the provided comparison. An interesting future research topic is making a comparison between the envelopment form of selecting models under different returns-to-scale assumptions.

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