Free Vibration Analysis of Rotating Functionally Graded Beams Using the p-Version of the Finite Element Method

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Abstract: *The functionally graded materials (FGMs) are inhomogeneous composite materials where the properties of FGM constituents vary gradually and smoothly. The smooth variation of the material properties overcomes the adverse effect of the laminate and sandwich composites structure such as the delamination mode of failure caused by the large interlaminar stresses. In this study, the free vibration of the lead-lag motion of a rotating functionally graded beam (RFGB) is investigated. From Hamilton's principle, the linear partial differential equations are derived for coupled stretching and bending motion. The governing equations based on Euler–Bernoulli beam theory accounts for centrifugal forces field, the centripetal acceleration and the gyroscopic effect. A p-version of the finite element method in conjunction with the modeling dynamic method using the arc-length stretch deformation is applied to find natural frequencies and modes shape of the cantilever beam. The displacements are expressed as the combination of the in-plane and out-of-plane shape functions, enriched with trigonometric hierarchical shape functions used generally to give additional degrees of freedom (dof) to the interior of the element. The convergence properties of the rotating beam Fourier p-element is examined, the results are compared with those of the literature where excellent agreements are observed. The influence of angular speed, Young's modulus ratio and power-law exponent on the natural frequencies and mode shapes is investigated. The tuned rotating speeds at which the beam will vibrate violently are determined for stainless steel–silicon nitride RFGB versus the power-law exponent.*

Keywords: *rotating functionally graded beam, Fourier p-element, lead-lag free vibration, gyroscopic effect, hybrid displacements, tuned rotating speed*

1. Introduction

There are many examples in mechanics which can be modeled as rotating cantilever beams, such as turbine blades, turbo-engine blades, helicopter blades, robot manipulators and wind blades. Knowledge of the vibrational behavior of these structures helps to avoid resonance and instability problems during their operational life. The functionally graded materials (FGMs) are composite materials where the microstructures are locally varied. The composition and the volume fraction of FGM constituents vary gradually, giving a non uniform microstructure with continuously graded macro properties. For instance, one face of a structural component may be an engineering ceramic that can resist severe thermal loading, and the other face may be a metal to maintain structural rigidity and toughness. The smooth variation of the material properties in the body overcomes the adverse effect of the laminate and sandwich composites structure such as the delamination mode of failure caused by the large interlaminar stresses. Details of design, processing and applications of FGMs can be found in Koizumi (1997) and Miyamoto and al. (1999).

The research work related to the modeling and behavior of rotating functionally graded beams (RFGB) made of FGMs has been limited to a few papers. Librescu and his co-workers (Oh *& al.*2003, Librescu *& al.*2005, Librescu *& al.*2008) are the first ones dealing with a rotating blade made of FGMs. The blade, mounted on a rigid hub, is modeled as a thin-walled beam that incorporates the warping restraint and the pretwist effects. Librescu and his co-workers have treated the problems of free vibration and thermoelastic modeling of turbomachinery thin-walled rotating blades made of FGMs and operating in a high temperature field. Fazelzadeh and Hosseini (2007) investigate the aerothermoelastic behavior of supersonic rotating thin-walled beams made

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